Conservation Laws, Symmetries, and Elementary Particles

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The following student text on conservation laws, symmetries, and elementary particles was developed in a Dutch project for teaching modern physics to the top stream of the sixth year of secondary education (age 17–18). In a series of 35 lessons of 45–50 minutes each, students study particle-wave duality, the Heisenberg principle, probability models for properties of particles, the particle in a box, and applications, elementary particles, and astrophysics (http://www.phys.uu.nl/~wwwpmn). In this paper we focus on particle physics and the key concepts of this chapter are: transformation, reaction equation, conservation laws, and symmetry. For recent literature regarding the teaching of symmetries and/or elementary particles, we refer to articles by Hill & Lederman,[1] Pascolini & Pietroni,[2] Kalmus,[3] O'Connell,[4] and Hanley.[5]

Instead of discussing a multitude of particles and reactions, the core of the elementary particle chapter is formed by a discussion of conservation laws and symmetries. Before getting to this point students have encountered reaction equations, nuclear reactions with a few examples, energy and mass, binding energy and mass defect computations, quarks and leptons, and accelerators and detectors. In the following we present the student text as an example of how to deal with elementary particles at the secondary level and as a handy background article for teachers.

The reasons for our focus on conservation laws and symmetries are:

a) The conservation laws provide a nice connection with the classical physics background of students.

b) A focus on conservation laws and symmetries matches the current emphasis in elementary particle physics and is useful in other branches of physics as well.

c) Using the laws and symmetries in reaction diagrams provides an opportunity for reasoning with main principles while an approach with lots of different particles (particle zoo) and reactions may present too many details, which will be forgotten anyway.

The use of the term conservation laws and symmetries might generate some expectations that we cannot fulfill. We expect our students to be able to apply symmetry principles in reaction diagrams and use these as a tool to determine whether or not reactions are possible and to predict alternative reactions. We do not expect students to fully understand the connection between symmetry and a conservation law.[1]

Because in an earlier Dutch project the use of Feynman diagrams in secondary physics courses was not successful, we use simplified diagrams and call them *reaction diagrams* rather than Feynman diagrams. The diagrams are only used to describe and predict reactions. They are not used to infer the probability of reactions or look deeper into the nature of the interactions.

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The Student Text

Conservation principles are at the core of reaction equations and in different walks of life one uses different terminology. In daily life one can say that house keys do not dissolve when it rains. In physics one could say that the number of keys is *conserved* in interactions with raindrops.

In many kinds of reactions the number of electrons is conserved. However, there is no absolute conservation law, because in nuclear reactions and in reactions of subatomic articles the number of electrons can change. For example, β^- decay produces an electron that was not there before:

$$n \rightarrow p^+ + e^- + \bar{\nu}_e \quad .$$

This is why instead of discussing the number of electrons, we introduce another number: the lepton number. Leptons (light particles) are electron-like particles and neutrinos specific for each kind and labeled ν_e, ν_μ, and ν_τ. The lepton number is defined as

lepton number = number of leptons minus the number of anti-leptons.

Because there is no known reaction in which the lepton number changes, we can say that the lepton number is *conserved*. As far as we know, this is an absolute conservation law, comparable to the conservation of charge.

Long before the theory of quarks, it was already noticed that a similar conservation law could be applied to particles like protons, neutrons, and other similar particles jointly called *baryons*. The conservation law applies to the so-called baryon number:

baryon number = number of baryons minus number of antibaryons.

Quark number can be defined as the number of quarks minus the number of antiquarks. Conservation of baryon number really boils down to conservation of quark number. At present we know that all baryons consist of three quarks, so their quark number is three, whereas mesons consist of a quark and an antiquark and have zero quark number. In all known reactions involving mesons and baryons, both baryon and quark numbers are conserved. Baryon number, however, is the more common term, because we already know it from nuclear physics. The mass number of a nucleus is actually the baryon number. The atomic number (*Z*), the number of protons in a nucleus, expresses the amount of positive charge and is sometimes called *charge number.*

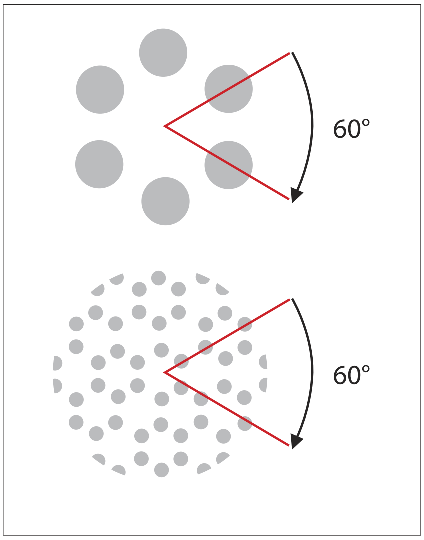

Fig. 1. Symmetry in graphite hexagon and in graphite lattice.

Symmetries

Earlier in this chapter we have already seen that conservation laws are important in analyzing reactions. Another closely related and convenient way of analyzing reactions is the use of symmetries. What is symmetry?

The crystal lattice of graphite provides a clear example of symmetry (see Fig. 1). It is symmetric with respect to rotation over 60°. The principle of symmetry is that there is a property, the pattern of the crystal lattice, which does not change under certain operations, in this case a rotation over 60°. Such a property is called a *symmetry property* and the operation is called a *symmetry transformation*.

We will deal with symmetry transformations for particle reactions. The symmetry *property* we study is whether a reaction is possible.[6] We then look at different symmetry transformations, each time following the same principle: We take an existing reaction equation, change something, and ask whether the result can also occur in nature.

Time Reversal and Charge Reversal Symmetry

Time reversal (T) is an operation that reverses a process in time (as distinguished from space). In other words, the arrow in the reaction equation is reversed. A reaction is symmetric under time reversal if the reverse reaction is also possible. The creation of an electron-positron pair is the reverse of the annihilation that occurs when an electron and a positron meet. Time reversal symmetry means that the arrow in the equation can be reversed. For example,

thorium-232 decays into radium-228 while emitting an alpha particle:

$$^{232}_{90}\mathrm{Th} \rightarrow {}^{232}_{88}\mathrm{Ra} + \alpha.$$

Collisions between radium and α-particles *of the proper energy* can result in the production of thorium:

$$T \;\Rightarrow {}^{232}_{88}\mathrm{Ra} + \alpha \rightarrow {}^{232}_{90}\mathrm{Th}.$$

The macroscopic world is clearly not time symmetric. When viewing a videotape you will notice within seconds whether it is run forward or backward. However, on a microscopic scale most reactions can be reversed. For a long time physicists thought that on a microscopic scale the world is indeed rigorously symmetric. The reversibility of time and the physics and philosophical questions related to that constitute an interesting problem, with many questions remaining.

A second and closely related symmetry is symmetry under charge reversal (C: Charge conjugation). With this we mean that particles are being replaced with their antiparticles. Symmetry under **C** means that the reaction equation remains valid if all particles are replaced by their antiparticles. For example,

A proton and an electron together form a hydrogen atom:

$$\mathrm{p}^+ + \mathrm{e}^- \rightarrow \mathrm{H}.$$

Based on this, one would expect that an antiproton and a positron together would form an antihydrogen atom:

$$C \Rightarrow \mathrm{p}^- + \mathrm{e}^+ \rightarrow \bar{\mathrm{H}}^-.$$

Since 2002 such antiatoms can be produced in considerable quantities at CERN in Geneva.

Diagrams

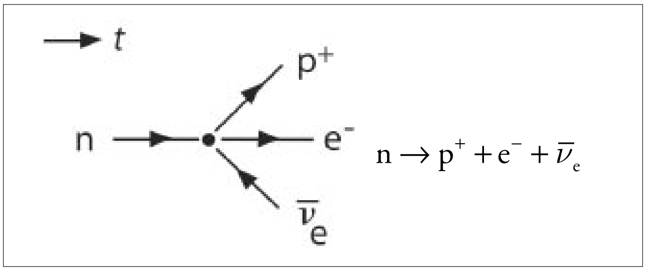

Fig. 2. Diagram of β^- decay.

Particle reactions can be visualized using diagrams which we will call reaction diagrams. In the diagram in Fig. 2, time is going from left to right. The lines stand for particles; points where the lines come together (called *vertex*) visualize interactions. The diagram expresses a conservation law: conservation of baryon number in the case of the proton and conservation of lepton number in the case of the electron and the antineutrino. The fact that in the diagram in Fig. 2 the arrow of the antineutrino points to the left means that the lepton number (-1) is opposite to that of the electron (+1). Also with a positron the arrow would be to the left (lepton number –1), just like with the antiproton (baryon number –1).

The diagrams of Fig. 3 visualize a pair creation-reaction equation and its time-reversed annihilation reaction. The symmetries can be formulated as rules for operations on diagrams. ***T***-symmetry means that the diagram can be flipped around[7] (Fig. 3).

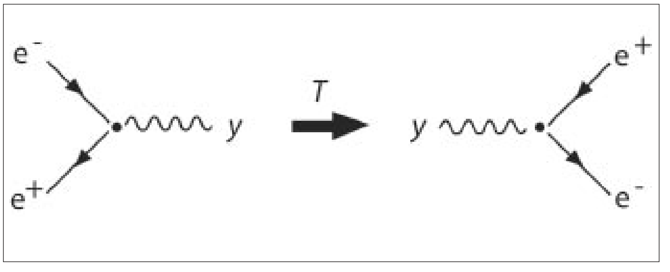

Fig. 3. Diagram of electron-positron annihilation and creation.

Electron Capture and β^+ Decay

β^- decay occurs in nuclei with a surplus of neutrons. In the nucleus a neutron is converted into a proton (see Fig. 2).

In nuclei with a relative shortage of neutrons, the reverse reaction can occur in which a proton is converted into a neutron. This result can be achieved through two different reactions.

The first of these reactions is called *electron-capture*: a proton and an electron can react, resulting in a neutron and a neutrino (Fig 4).

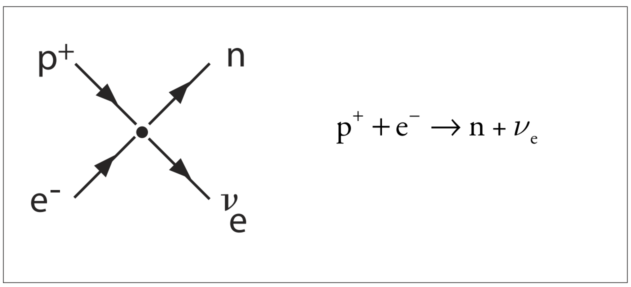

Fig. 4. Diagram of electron capture.

This reaction takes place primarily in heavy nuclei. The inner electrons are then close to the nucleus, which increases the chances of electron capture.

If electron capture is not possible or has a low probability, then it is still possible that a proton decays into a neutron while emitting a positron and a neutrino.

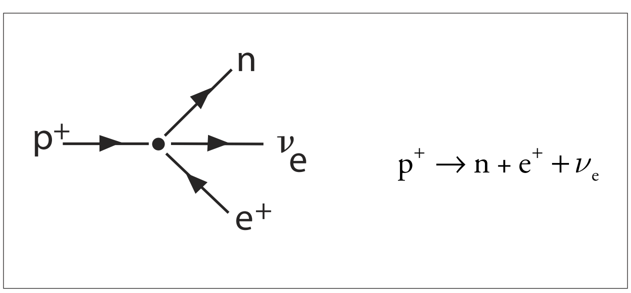

Fig. 5. Diagram of β^+ decay.

This reaction cannot take place in a free proton as the reaction requires energy. Within a nucleus such energy might be available if there is a surplus of protons. In the nucleus neutrons experience only attractive nuclear forces of protons and other neutrons.[8] However, protons experience attractive nuclear forces as well as repulsive electrostatic forces of other protons. If there are too many protons the nucleus might become unstable due to the electrostatic potential energy. Some of the electrostatic potential energy is used to create mass when a proton decays into a neutron and a positron plus a neutrino (Fig. 5). The last two will be ejected from the nucleus. The reaction is called β^+ decay. On the other hand, the decay of a neutron into a proton plus an electron and neutrino is called β^- decay.

Crossing

Time reversal means that the reaction arrow is reversed. Charge reversal, also called *charge conjugation*, means that all particles that participate in the reaction are changed into their antiparticles. A comparison of the diagrams for β^- decay, β^+ decay, and electron capture (Figs. 2, 4, and 5) suggests yet another symmetry that could be applied to the *individual* particles in a reaction. The operation that is needed to relate the different reactions is a combination of T and C for the separate lines in a diagram.

The symmetry operation in which an individual particle is taken from one side of the reaction arrow to the other side and then is converted in its antiparticle is called *crossing.*

We will use the symbol X for this operation. It turns out that crossing symmetry is indeed valid for every kind of particle reaction. In a reaction diagram this means that any line can be flipped over to the other side (or mirrored with respect to a vertical mirror) in which the arrow still points in the same way with respect to the vertex (toward vertex or away). Figure 6 shows this more clearly. In the reaction on the left, a neutron decays into a proton, an electron, and an antineutrino. In the reaction on the right, a neutron reacting with a neutrino produces a proton and an electron.

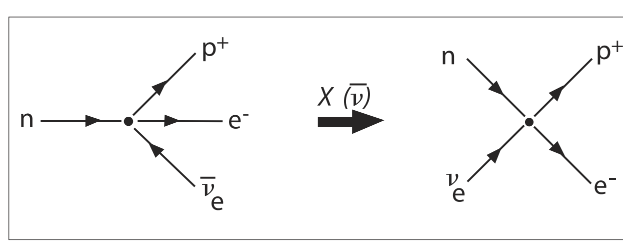

Fig. 6. Through crossing in the β^- decay diagram the possibility of neutrino capture by a neutron can be predicted.

The reactions for β^+ decay and for electron capture can be deduced from the reaction for β^- capture through a combination of crossing and time symmetry:

$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$

$X(\nu_e)$ $$n + \nu_e \rightarrow p^+ + e^-$$

T $$p^+ + e^- \rightarrow n + \nu_e.$$

and

$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$

$X(e^-, \nu_e)$ $$n + e^+ + \nu_e \rightarrow p^+$$

T $$p^+ \rightarrow n + e^+ + \nu_e.$$

Many other reactions can be obtained in a similar way, such as

$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$

$X(e^+)$ $$n + e^+ \rightarrow p^+ + \bar{\nu}_e.$$

All these reactions are indeed possible if the energy is available. The reaction on the right-hand side in Fig. 6, for example, means that neutrinos can cause nuclear reactions. This is the basis for the detection of neutrinos!

For physicists, diagrams have a deeper meaning than the mere representation of reaction equations. Particle physicists use so-called Feynman diagrams, which are constructed according to certain rules so that every line and every point has a mathematical meaning. From these diagrams they can compute the probability of a certain reaction taking place.

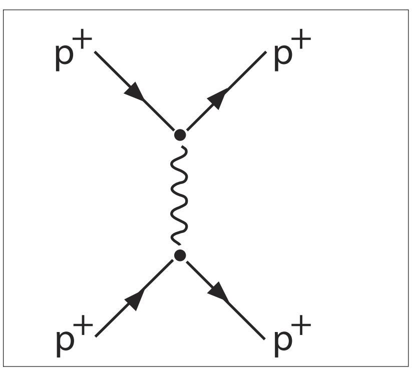

Fig. 7. Feynman diagram for one of the terms contributing to proton-proton scattering.

Feynman diagrams are also used to compute *forces.* Figure 7 gives a first approximation of the electrical force between two protons. Two protons approach each other from the left. They exchange a photon as part of the repulsion process. The angle between the incoming and outgoing proton can be used to compute the electric repulsion force.

For a long time, particle theories encountered major problems. Many of these problems originated from infinities in the computations based on the diagrams. For several decades beautiful results were followed by incomprehensible riddles and vice versa. Only toward the end of the 20th century were some of these problems solved, which led to the "standard model" of elementary particles. The Dutch physicists Gerard 't Hooft and Martinus Veltman received a Nobel Prize for their contributions to solving the infinities.

The formulation of the standard model does not mean that all problems have been solved. Although the predictions of the model match spectacularly with the experimental results, the theory still has some characteristics that physicists do not like. Also some important elements are still lacking, such as a good description of gravity. So far physicists have not been able to integrate quantum physics and gravity in one theory in spite of major efforts, although a tentative hypothesis called "string theory" has made encouraging progress in this direction.

The student text then ends the particle chapter with a paragraph about hadrons and quarks, and of course there are exercises (not included here). The pion problem (Appendix I)[9] shows the "style" of examination problems common at the pre-university level in the Netherlands.

Experiences

The student text has been used for two years now. The teachers are generally happy with the approach and the topic conservation laws/symmetries is considered one of the easier topics of the rather demanding modern physics course. Teachers who had tried the previous Dutch approach with Feynman diagrams and many more particles feel that the current approach with the simplified diagrams gives them a much better understanding and confidence. Some students are able to understand the topic by reading the text only; however, most students and teachers need a few additional examples and exercises to get the motivating experience of recognizing how neatly the diagrams can represent the reaction equations and how one can predict new reactions by applying the symmetries. Teachers typically spend two lessons on the topic. We suspect that some teachers now find the topic easy and go too fast with too little exercise as other topics in the course are considered more demanding. On the pion problem (Appendix I)[9] 75 students from seven classes/teachers scored 52% on the first question, 87% on the second, 68% on the third, and 54% on the fourth. The performance on the fourth question on diagrams ranged from 25%– 83% for different teachers. That range may show the difference between classes that include sufficient exercises and those that do not.

References

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4. J. O'Connell, "Comparison of the four fundamental interactions of physics," *Phys. Teach*. **36**, 27 (Jan. 1998).
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6. In advanced particle physics, physicists not only look at whether a reaction is possible, but also compute the probability that it takes place.
7. Pair creation only takes place near heavy nuclei, which absorb part of the momentum of a photon. Otherwise there would be no simultaneous conservation of energy-mass and momentum. For example, consider a photon that has just enough energy to create the mass positron and an electron. If energy is just enough, then the photon has momentum but the positron and electron will be at rest. Momentum conservation in this reaction is only possible if a nucleus nearby absorbs the momentum of the photon. Therefore, pair creation cannot take place in vacuum.
8. At extremely short distance, nuclear forces are repulsive to prevent collapse of the nucleus.
9. See EPAPS Document No. E-PHTEAH-43-006505 for Appendix I. This document may be retrieved via EPAPS homepage (http://www.aip.org/pubservs/epaps.html) or from ftp.aip.org in the directory epaps. See the epaps homepage for more information.

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Dick Hoekzema *and* **Gert Schooten** *were the developers of the Modern Physics Project. Both combine jobs in high school physics teaching with work in the project. Dick's prior experience with modern physics includes a Ph.D. dissertation on the "Foundations of Quantum Mechanics" while Gert specialized in condensed matter physics.* **Ed van den Berg** *recently joined the project after returning from a physics teacher education project in the Philippines.* **Piet Lijnse** *is professor of physics education at the University of Utrecht and director of the project.*

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